Data driven synthetic wavefront generation for boundary layer data

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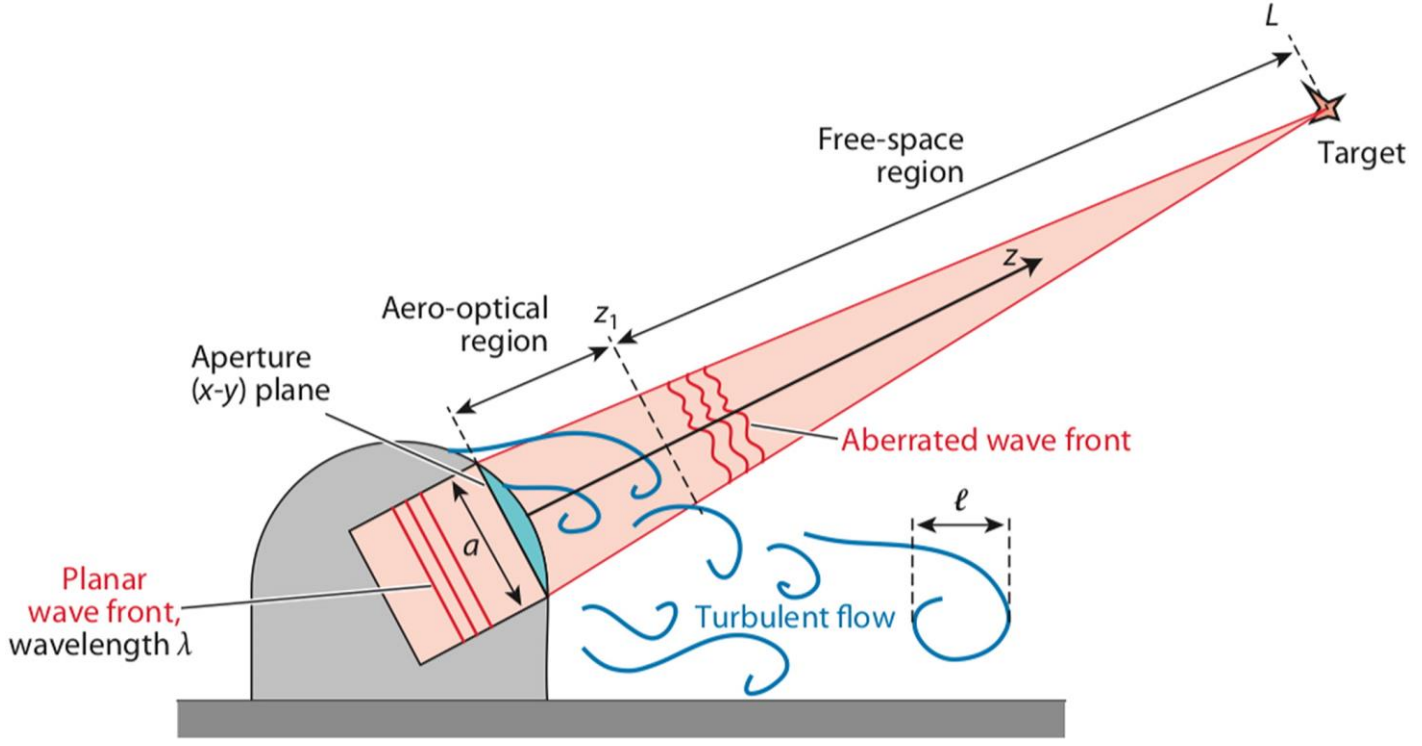
The Aero-Optics Problem

A turbulent flow field causes wavefront aberrations



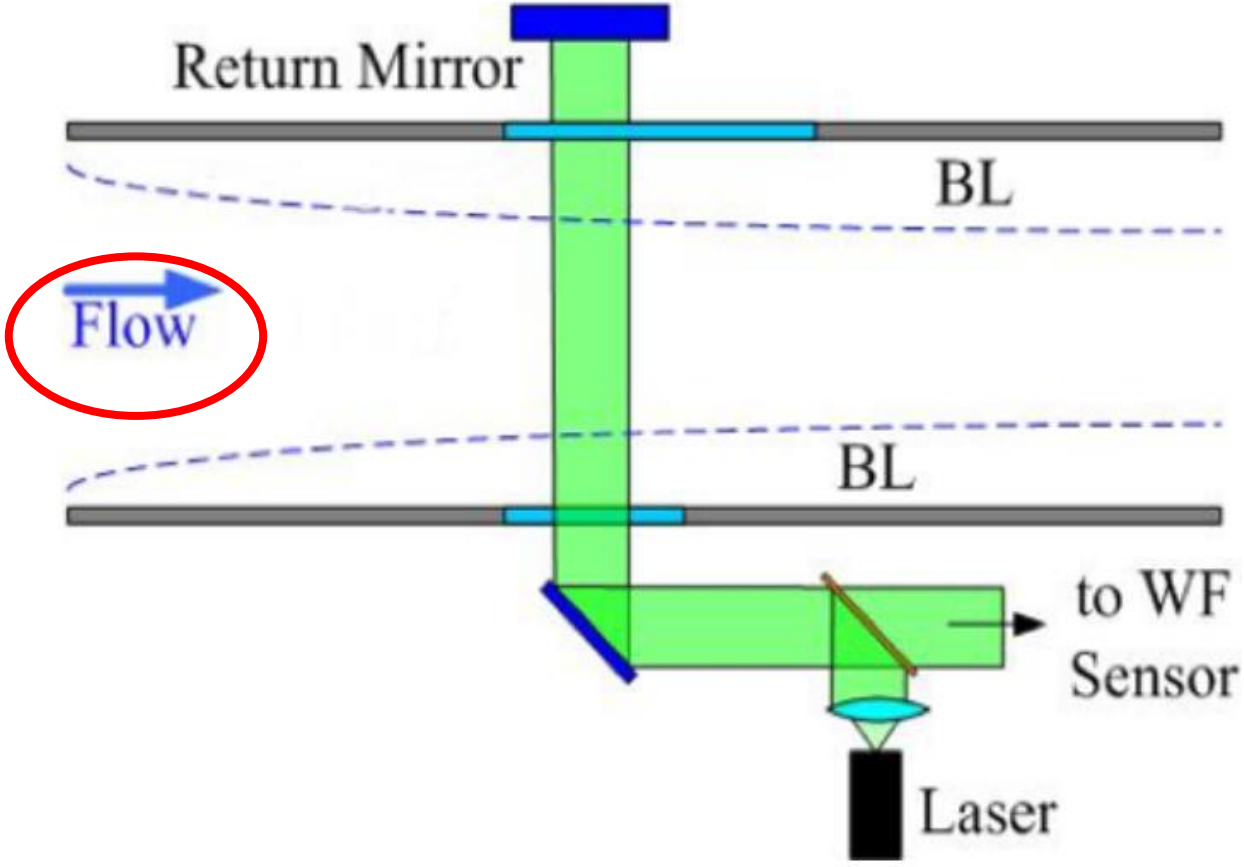
The Aero-Optics Problem, Figure 1 from [1]

[1] M. Wang, A. Mani, and S. Gordeyev, "Physics and Computation of Aero-Optics," *Annual Review of Fluid Mechanics*, Vol. 44, No. 1, 2012, pp. 299–321.

Adaptive Optics for Aero-Optical Effects

- Adaptive-optic (AO) systems compensate for wavefront aberrations.
- AO system development *requires wavefront aberration data* for testing.
- **Problem**: Existing data acquisition methods are *expensive and time-intensive*.
- **Solution**: We develop **ReVAR (Re-whitened Vector Auto-Regression)** to generate *synthetic aero-optics wavefront data*.
  - ReVAR captures the *spatial and temporal* statistics of input data on **arbitrary time-scales**.

Experimental Boundary Layer Data



Notre Dame Wind Tunnel Experiment, Figure 1(b) from [3]

[2] M. R. Kemnetz and S. Gordeyev, "Optical investigation of large-scale boundary-layer structures",
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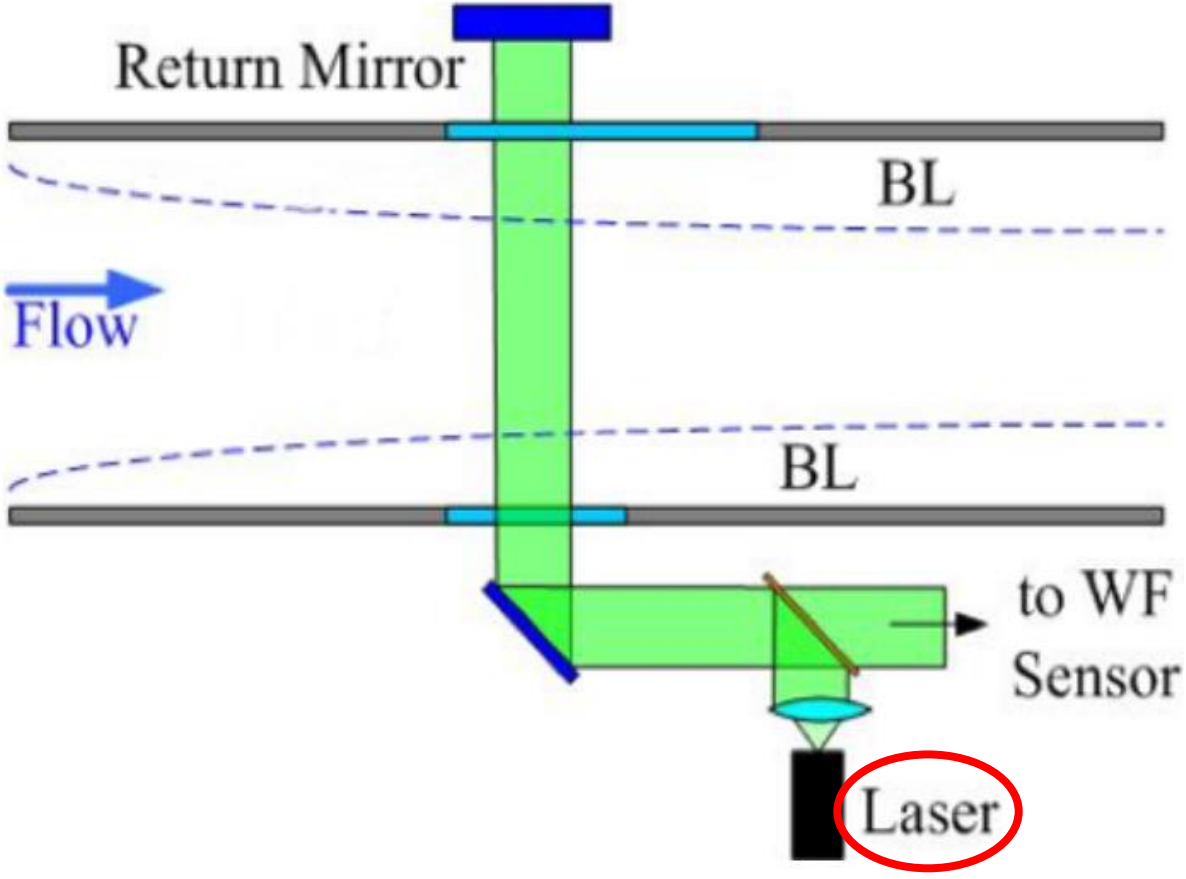
Experimental Boundary Layer Data



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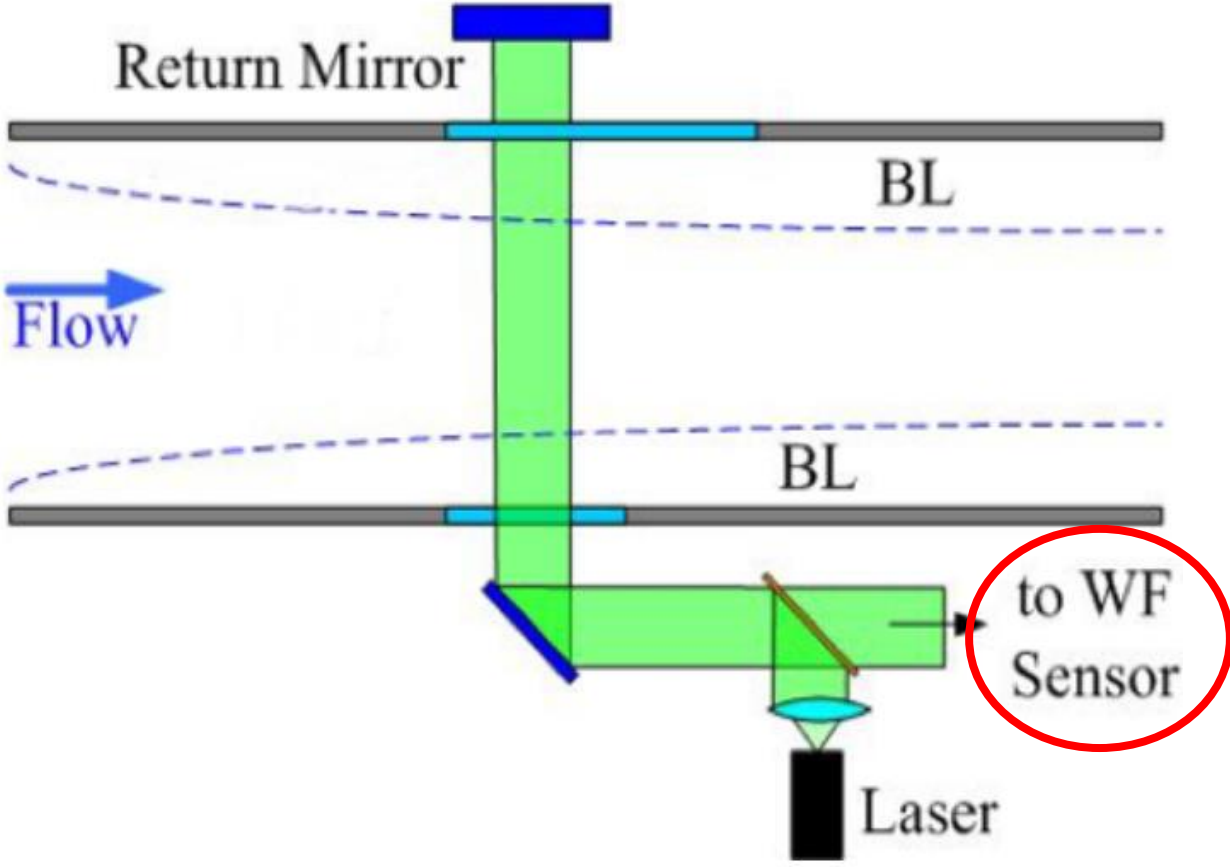
Experimental Boundary Layer Data

A **Shack-Hartman Wavefront Sensor** calculated optical path difference (OPD)



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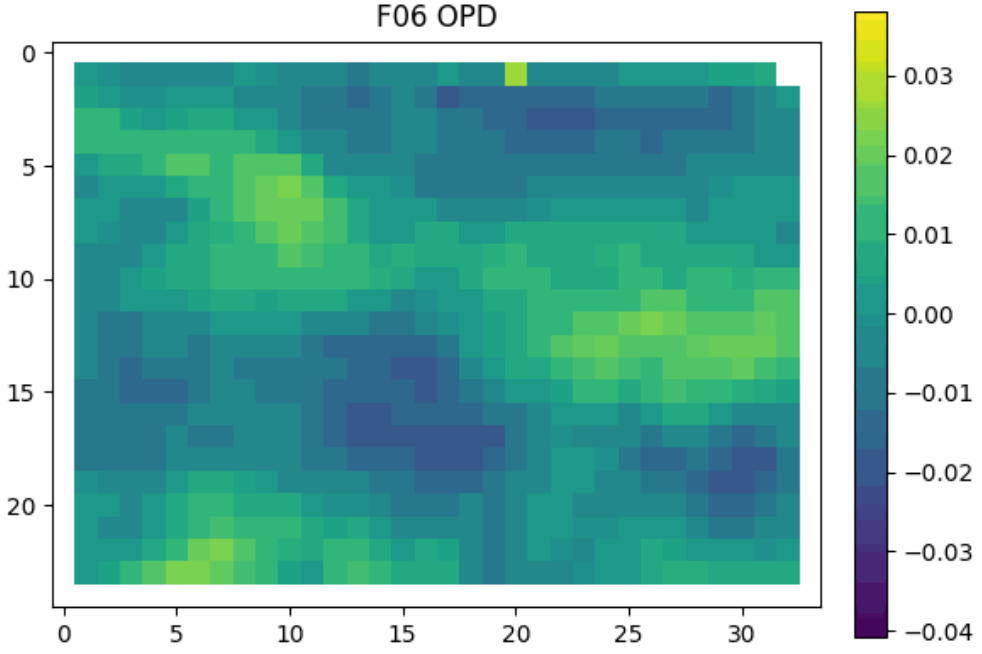
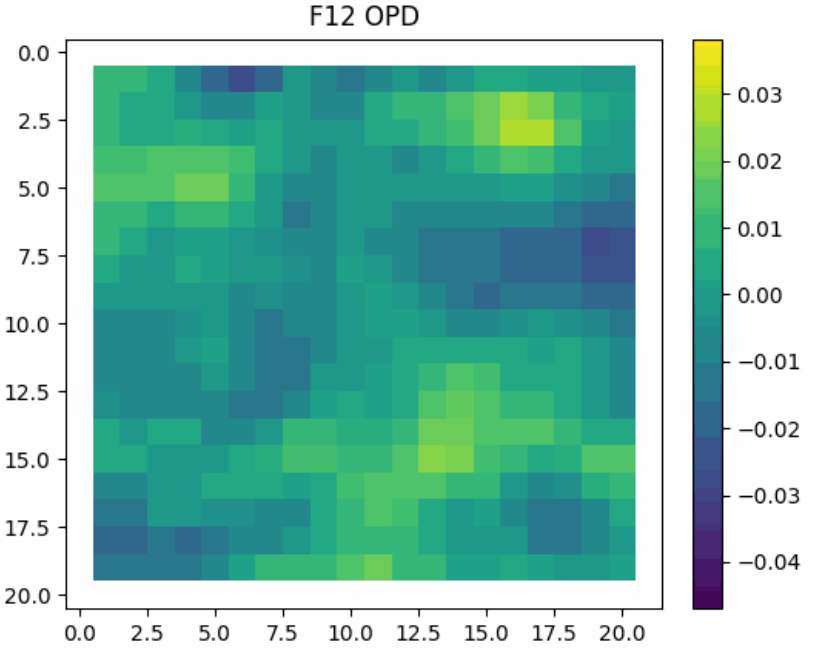
Experimental Data: Visualization





ReVAR: Overview

1. Data Analysis

2. Synthesis

3. Post-processing

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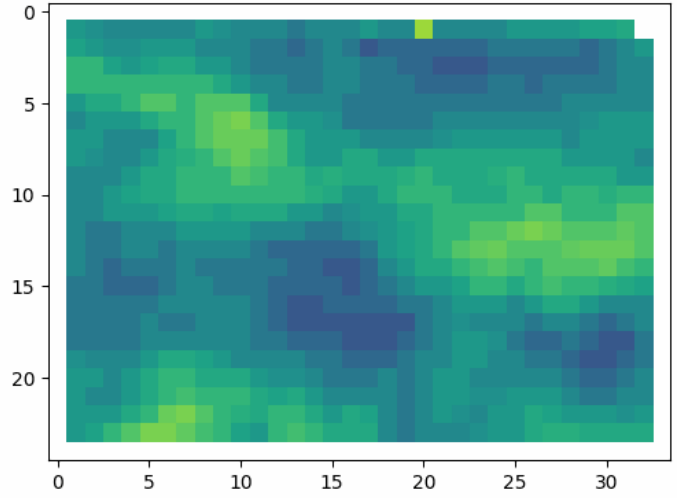
Data Analysis: PCA

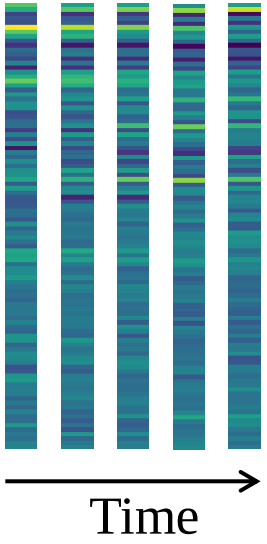
Spatially *de-correlate* the data using PCA.

Experimental Data X_n



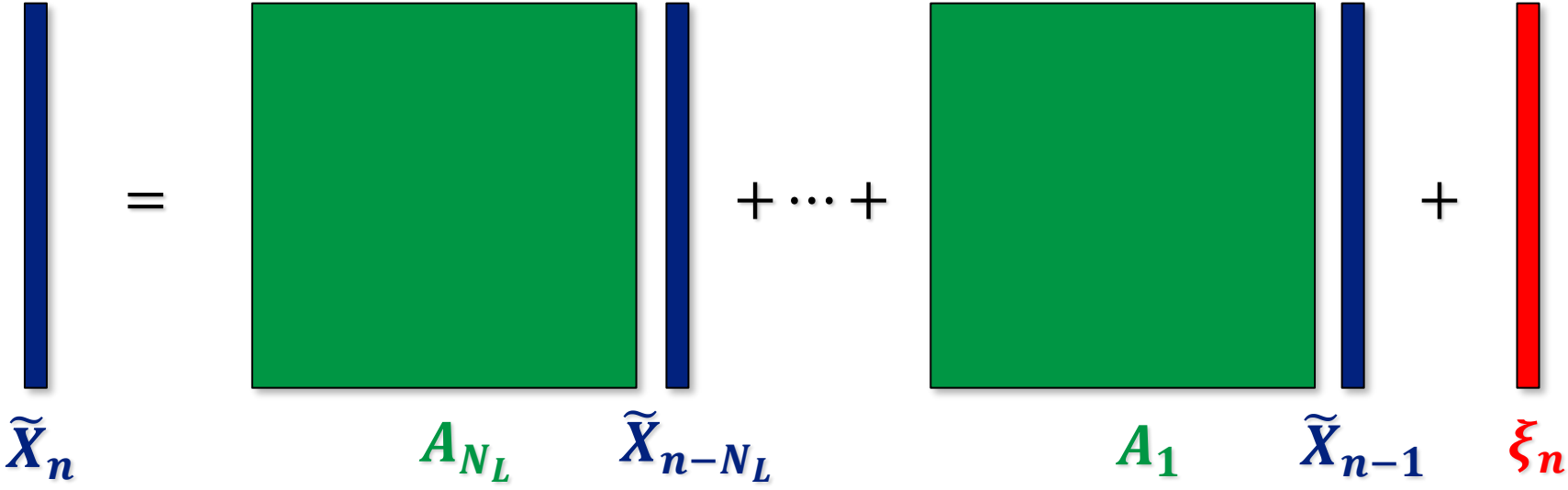
Spatial PCA

$\tilde{X}_n$



ReVAR: Vector Auto-Regression (VAR)

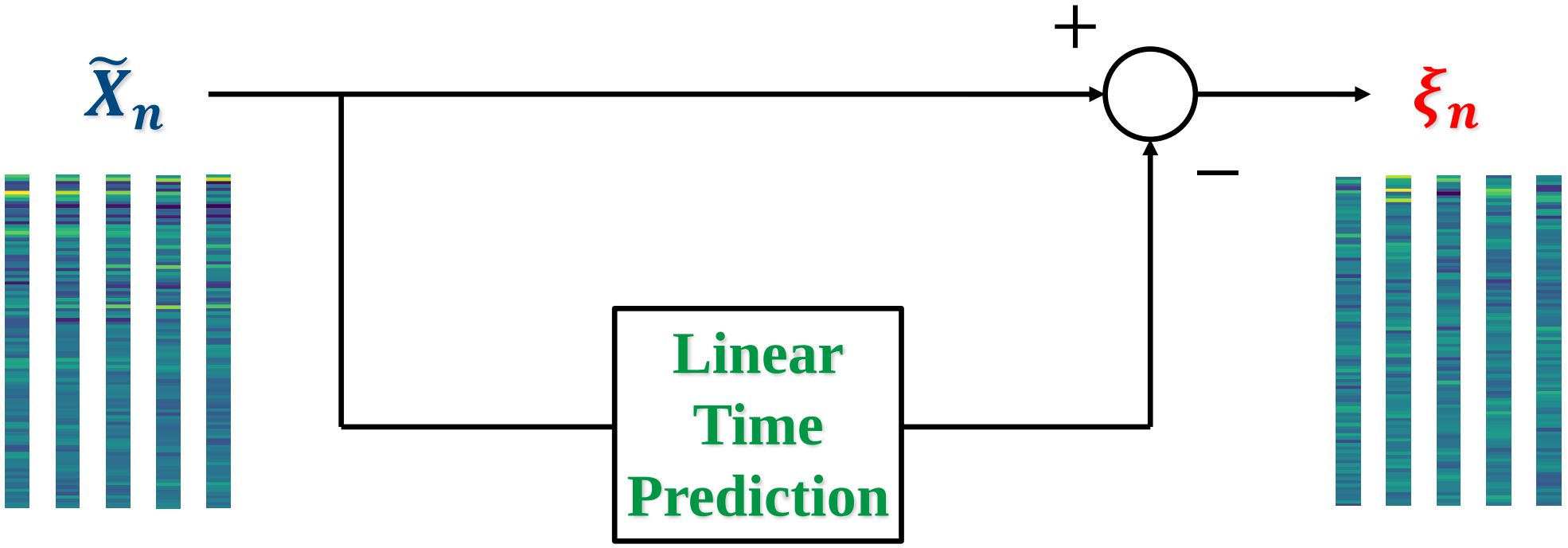
$$\tilde{X}_n = \underbrace{A_{N_L}\tilde{X}_{n-N_L} + \cdots + A_1\tilde{X}_{n-1}}_{\text{Linear Time Prediction}} + \underbrace{\xi_n}_{\text{Prediction Error}}$$



[3] H. Lütkepohl, *New Introduction to Multiple Time Series Analysis*, 2005, Springer-Verlag Berlin, pg. 13-14, 69-71.

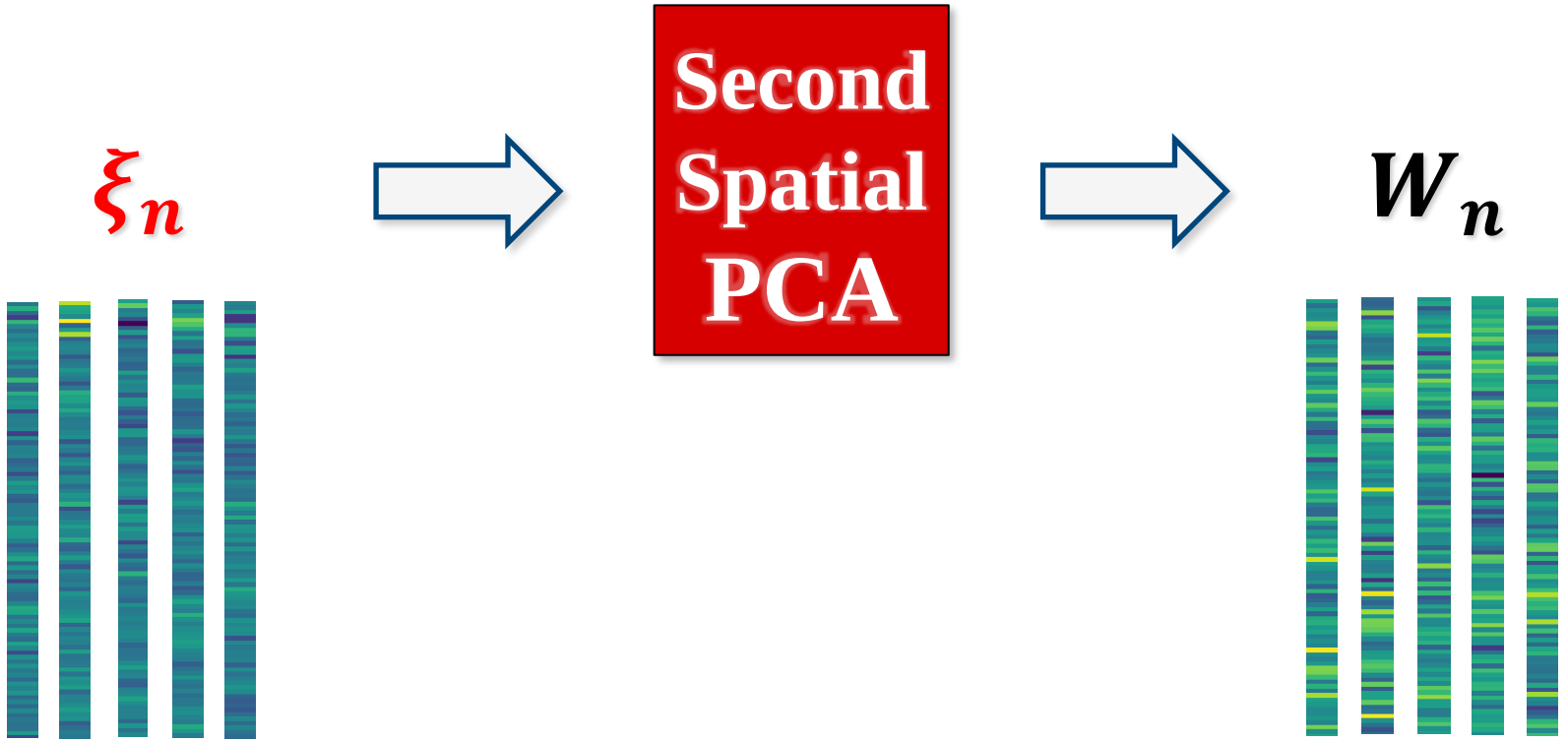
Data Analysis: Linear Time Prediction

ξ_n is **white** in *time* but **correlated** in *space*

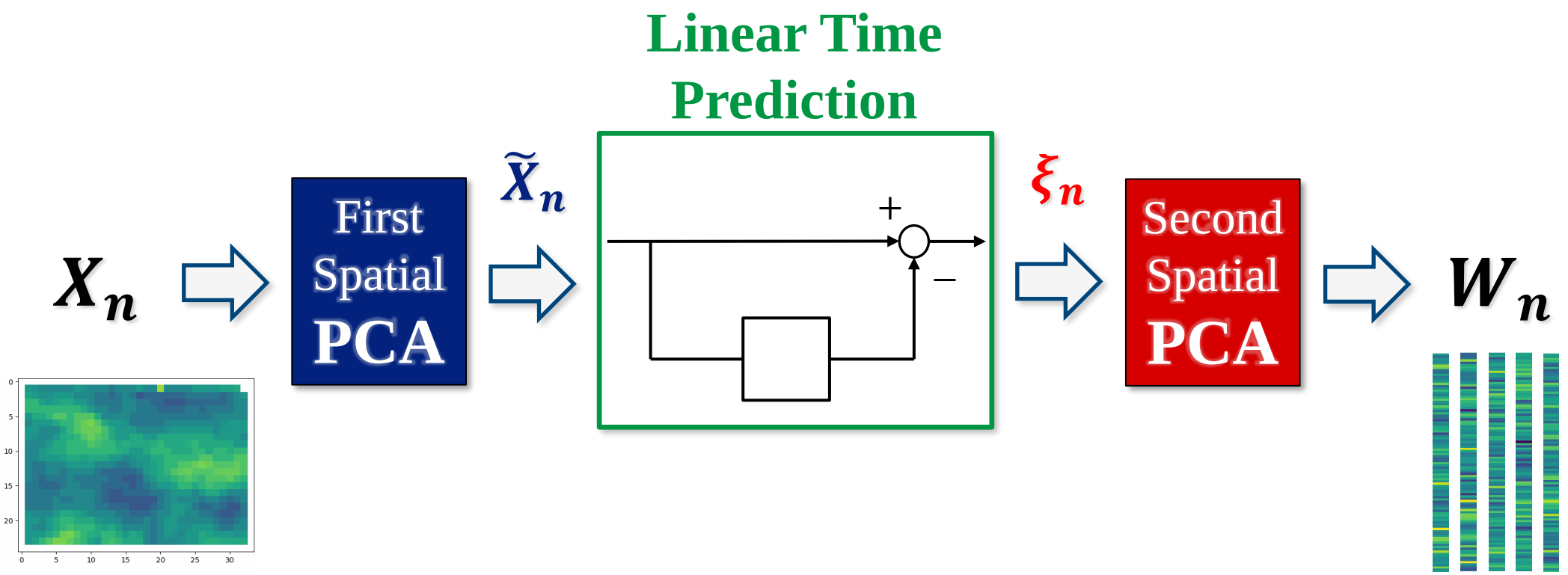


Data Analysis: Re-whitening

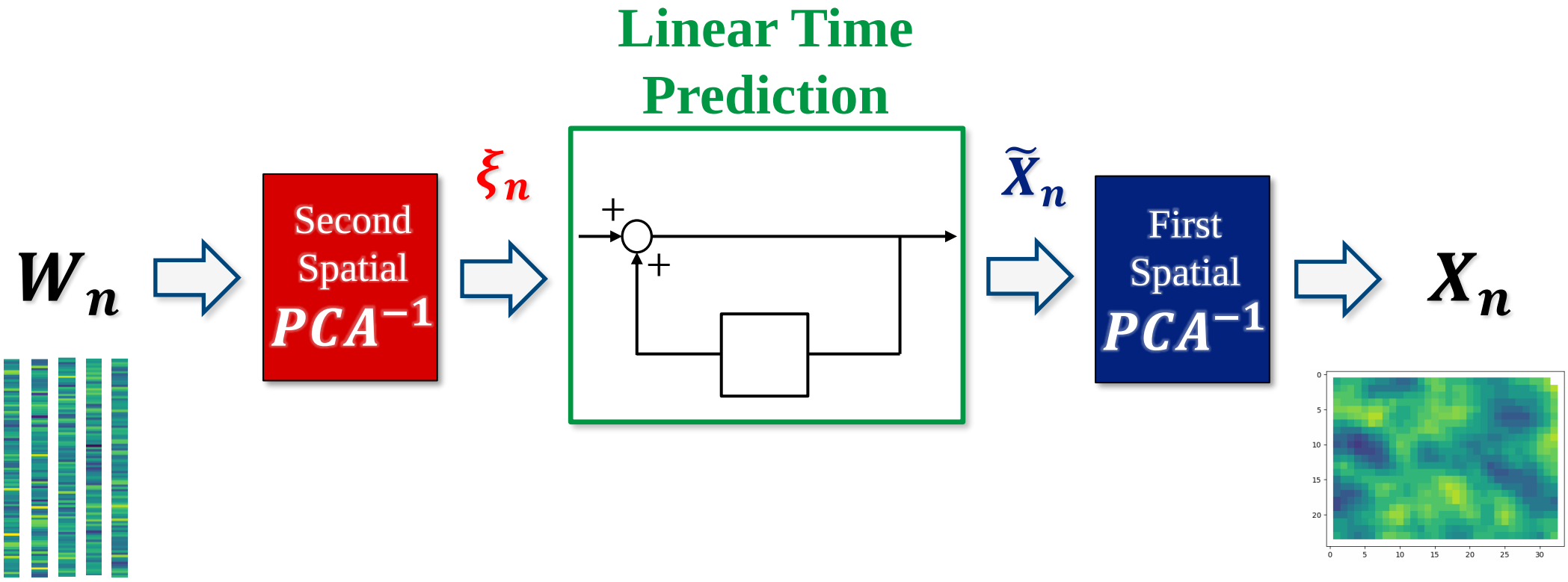
A **second PCA** *whitens* ξ_n in *space*



ReVAR: Data Analysis



ReVAR: Synthesis



ReVAR: Overview

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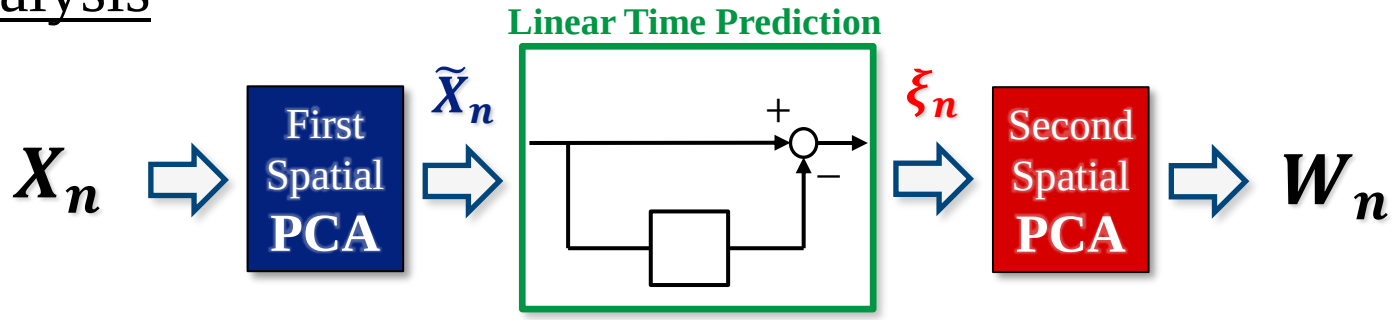
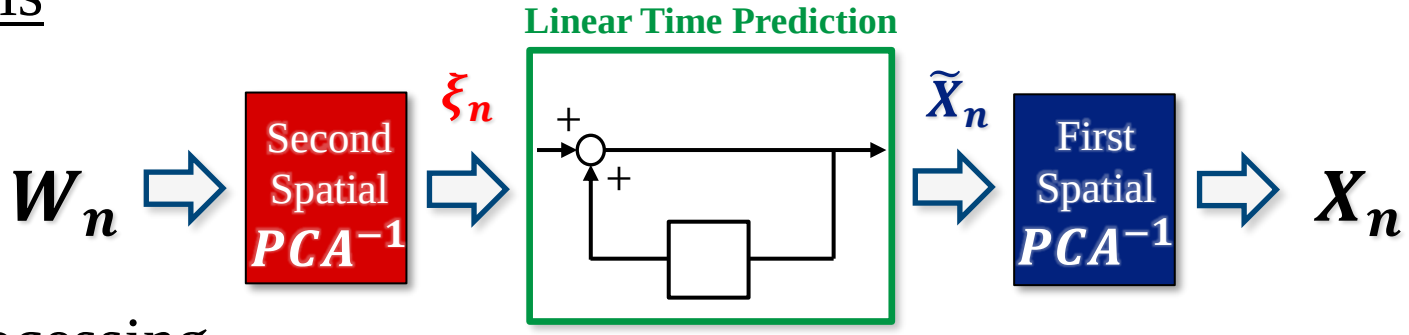


2. Synthesis



3. Post-processing

ReVAR: Overview

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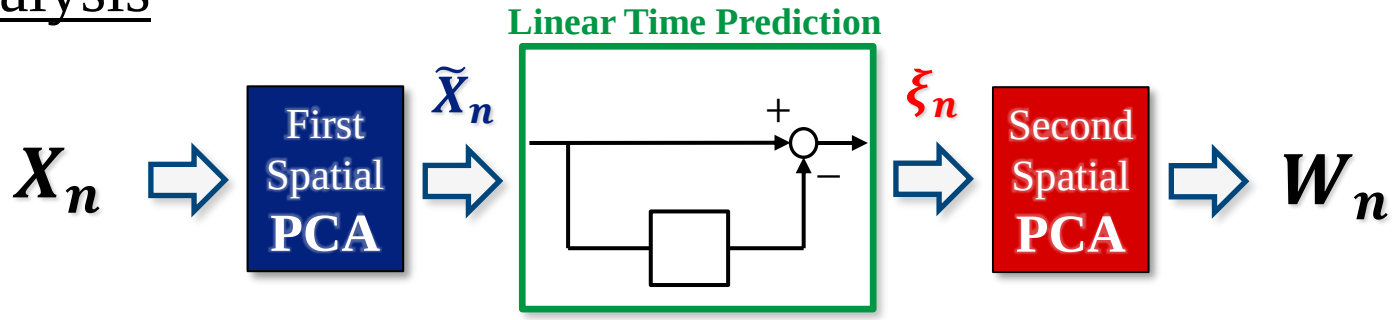

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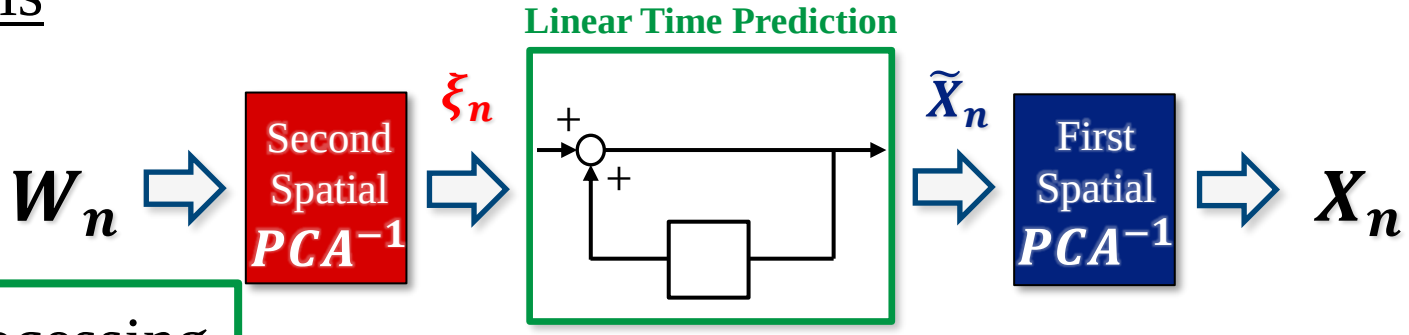
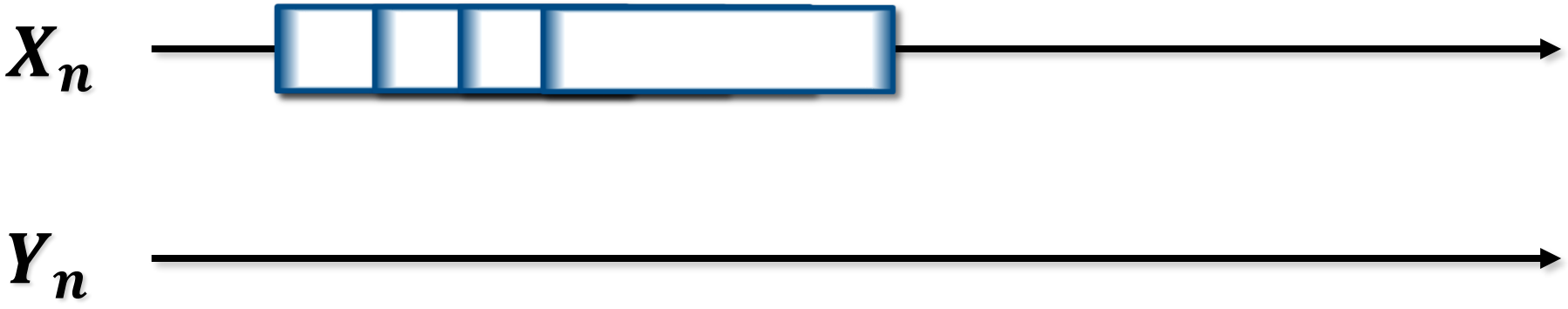


3. Post-processing

Post-Processing Synthetic Data

After synthesis, subtract a *(weighted) moving average*

$$Y_n = X_n - \sum_k X_k h_k \qquad \left(\sum_k h_k = 1\right)$$



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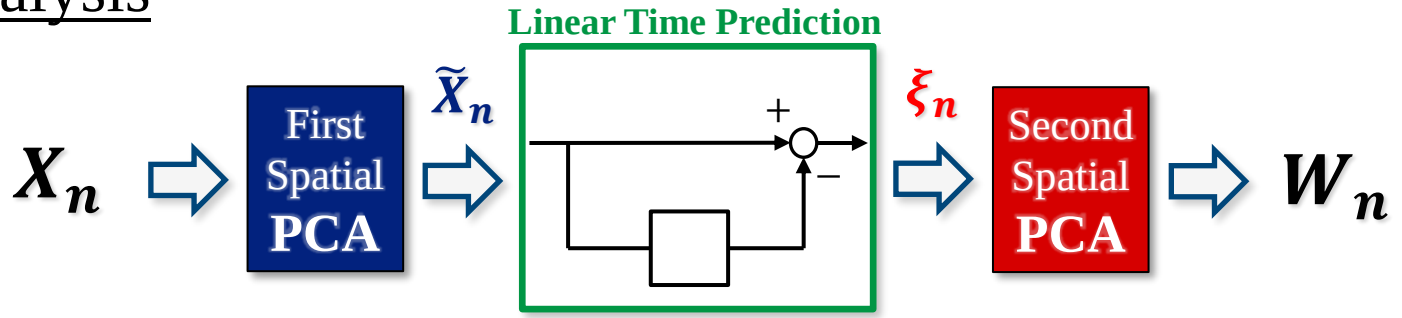
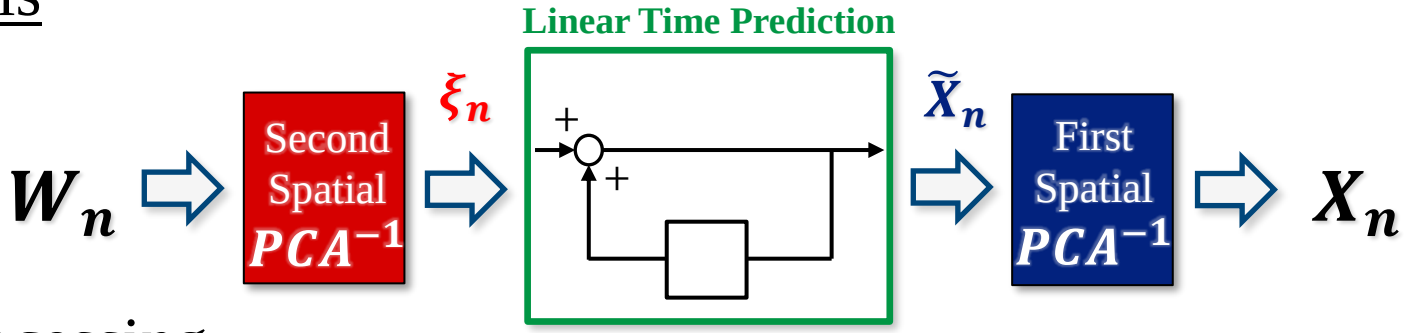
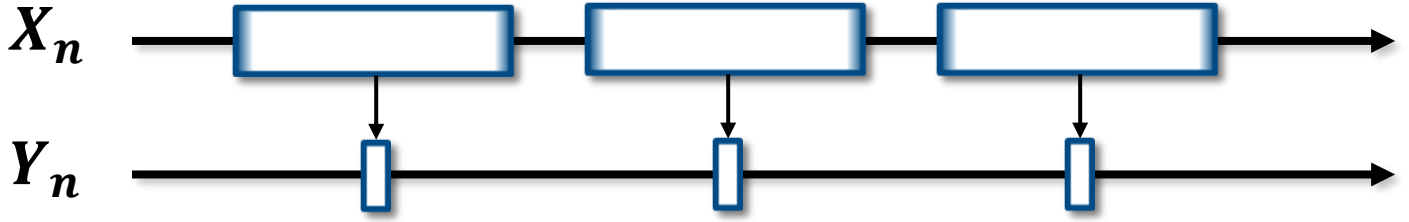


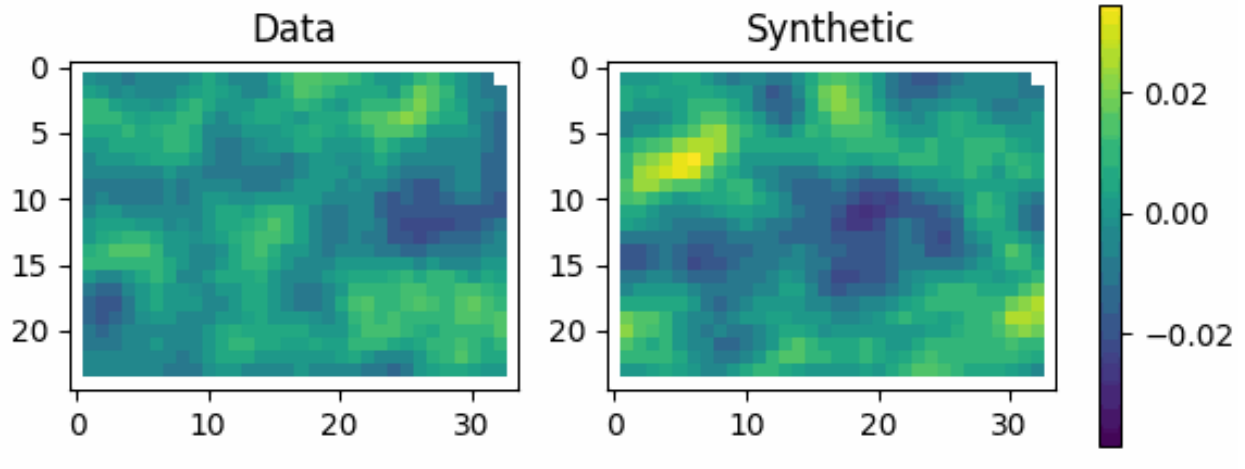
2. Synthesis

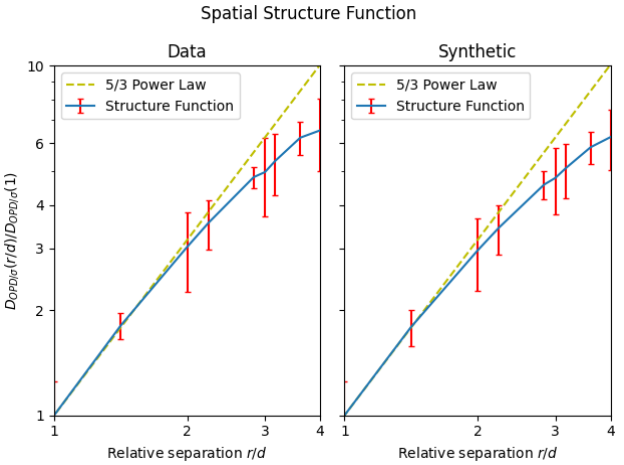


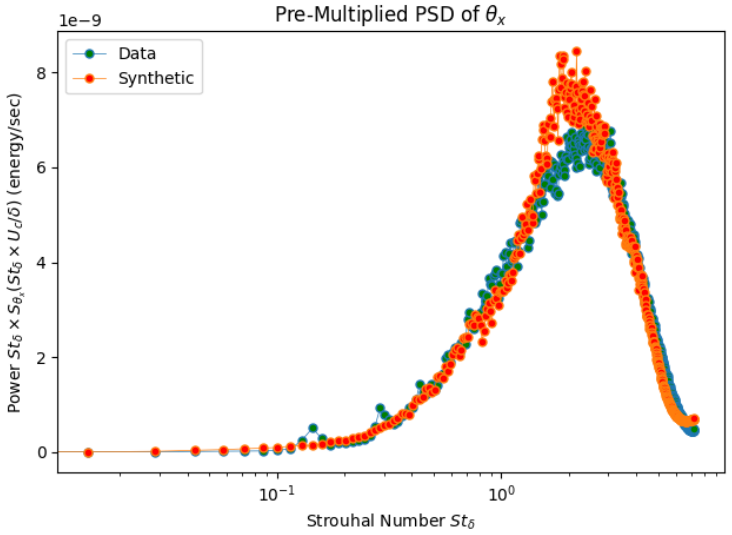
3. Post-processing

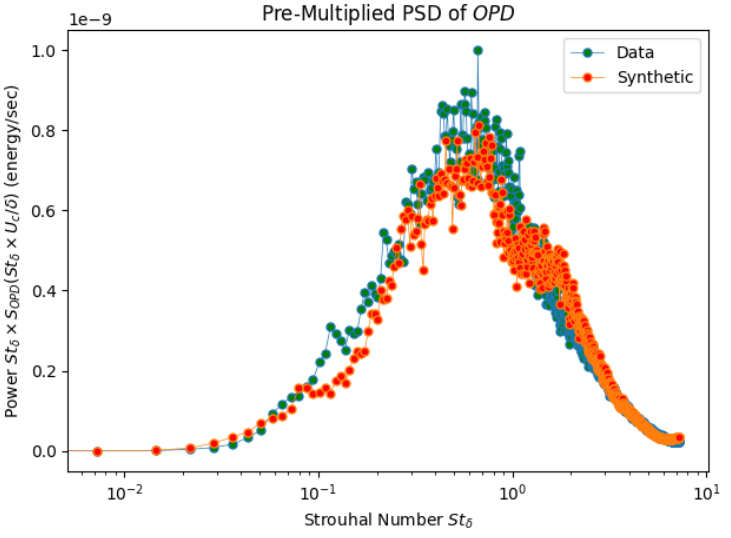


Results: Data Set F06 $(N_L = 3)$

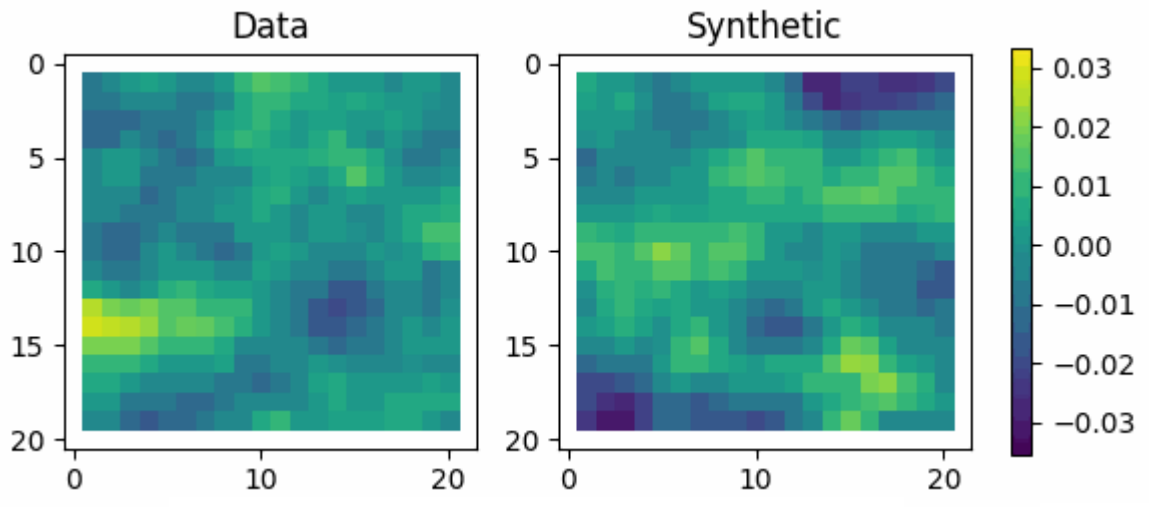


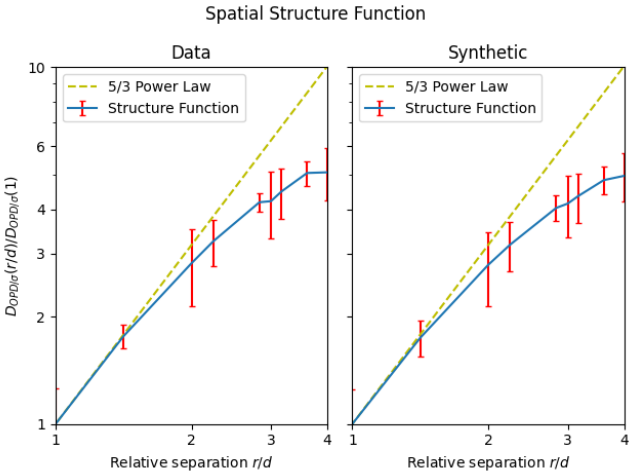


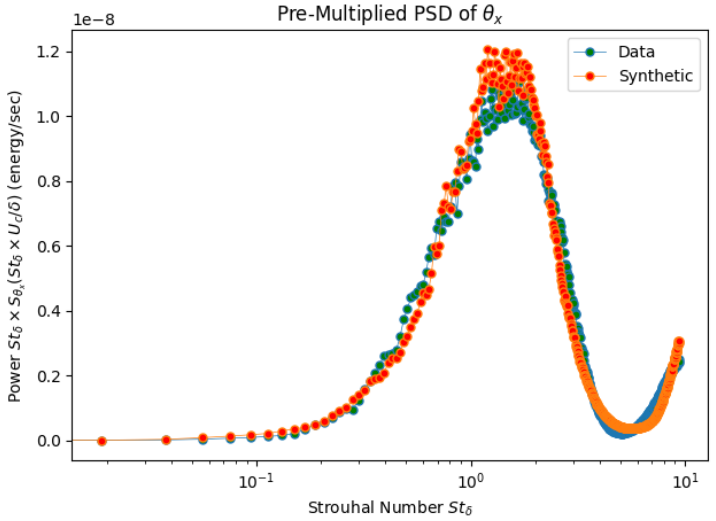


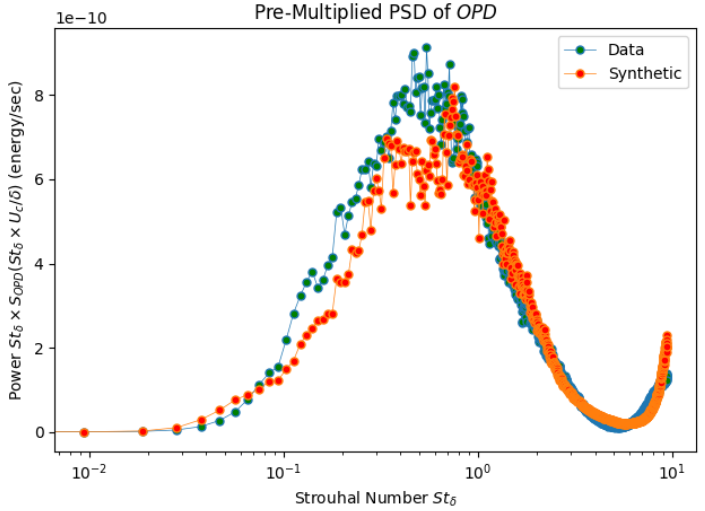


Results: Data Set F12 $(N_L = 3)$









Algorithm Run-Time

Data Set F06

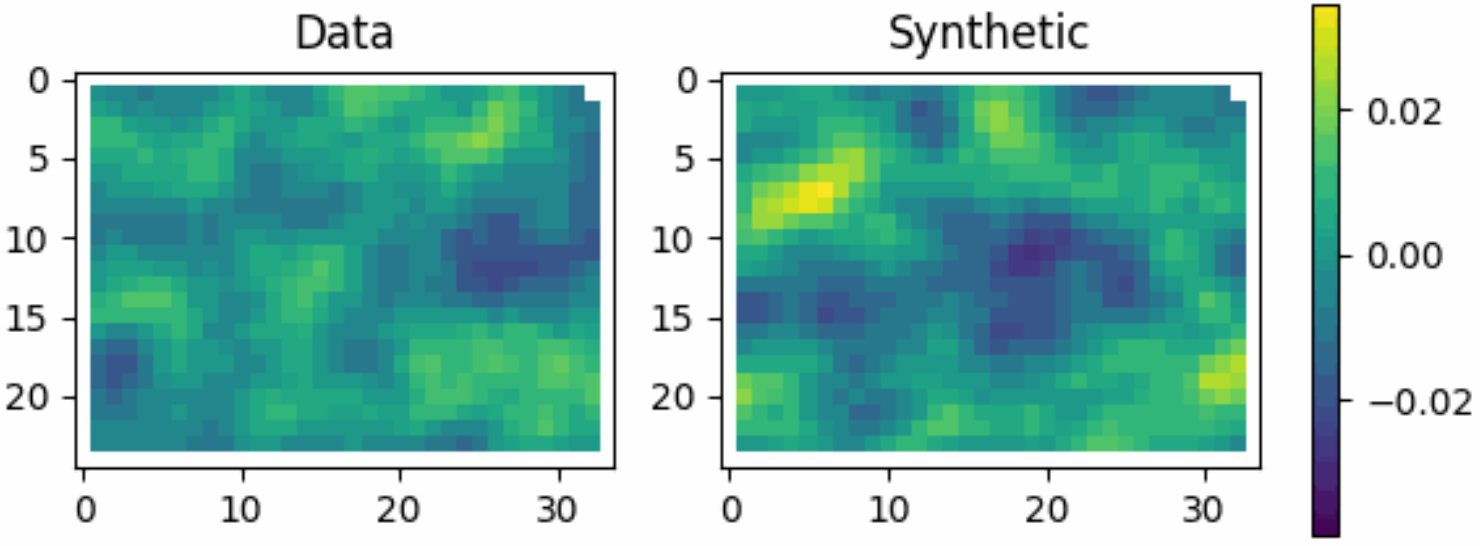
- Data Analysis: **3.8 mins**
- Synthesis & Post-Processing: **66 sec**
  (1 sec of synthetic data)

Data Set F12

- Data Analysis: **3.3 mins**
- Synthesis & Post-Processing: **38 sec**
  (1 sec of synthetic data)

Conclusion

- AO system development for aero-optics requires wavefront aberration data.
- Existing data acquisition methods are expensive and time-intensive.
- ReVAR is computationally efficient, runs quickly, and generates high quality synthetic wavefronts.



References

[1] M. Wang, A. Mani, and S. Gordeyev, “Physics and Computation of Aero-Optics,” *Annual Review of Fluid Mechanics*, Vol. 44, No. 1, 2012, pp. 299–321.

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